REPORT ON
THE SCIENCE ACADEMIES’ LECTURE WORKSHOP ON “ANALYSIS”
10-12th AUGUST 2016 AT MIRANDA HOUSE, DELHI

The Science Academies’ Lecture workshop on “Analysis” was organized at Miranda House, UNIVERSITY OF Delhi, Delhi from 10-12th August 2016. It was attended by 115 students, researchers and teachers. The three experts were Prof. Gadadhar Misra, Prof. B. Ramakrishnan, Dr. Gautam Bharali. Prof. Ajit Iqbal Singh is the convener and Dr. Daulti Verma is the co-ordinator. Each expert delivered three lectures followed by the interactive session. Towards the end of each day there was a tutorial, which was the most awaited session for the students. For our third year students it was the first experience to interact with such reputed mathematicians. The workshop ended with a great success on the third day. It was a wonderful learning experience for both students and teachers. The details of the lectures are given below in item A and a summary of feedbacks in item B.

A : Academic Programme.

The workshop started with Saraswati Vandana and a welcome address by Dr. Manjari Srivastava, a senior teacher in our department. The Chief Guest, Prof. V Ravichandran, Head, Department of Mathematics, University of Delhi addressed the students in the workshop. The workshop was attended by 90 students and from Miranda House and other colleges and 25 researchers and teachers.

(i) LECTURES BY PROF. GADADHAR MISRA ON “INEQUALITIES IN COMPLEX ANALYSIS”

The content of his lectures, though basic and self-complete, were new even for some teachers. He started his lecture by introducing the following concepts: Complex variables (Conjugate complex variable), Real analytic polynomials in complex variables $z$ and $\bar{z}$, Trigonometric polynomials. He then proceeded with the properties of these real analytic and trigonometric polynomials in complex variables like

(i) how we do differentiation of such polynomial,
(ii) when such polynomials are positive.

He explained how to construct such "Positive" polynomials in $z$ and $\bar{z}$ and gave some examples as well. He was mainly interested in the function $((log p)(z, \bar{z}))$. This function is defined only when the polynomial $p(z, \bar{z})$ is positive. That was the reason why he focused on the positivity of these real analytic polynomials. He then told about the partial derivative of the above function with respect to $z$ and $\bar{z}$.

He next explained: "When is this partial derivative with respect to $z$ and $\bar{z}$ positive?". To do so, he introduced the "Cauchy-Schwarz Inequality", concepts of linear transformations and diagonal linear transformations. On the last day, he proceeded with his earlier posed question i.e., "When is the partial derivative of the function"$(log p)(z, \bar{z})$" with respect to $z$ and $\bar{z}$, positive". He answered this question after constructing a positive polynomial $p(z, \bar{z})$ using the "Reproducing Property" of inner product space of polynomials and Cauchy-Schwarz inequality. He then also told about "Nilpotent Linear Transformation" and "Gram-Schmidt Orthonormalisation Process" to construct an orthonormal basis and hence found the matrix representation of this nilpotent linear transformation with respect to the already constructed orthonormal basis. This was to find an expression for the partial derivative of "$(log p)(z, \bar{z})$" with respect to "$z$" and "$\bar{z}$", and that expression came out to be a positive polynomial in $P(n)$. In this way he answered his question: "When is the partial derivative of the function $(log p)(z, \bar{z})$ positive for given polynomial, $p(z, \bar{z})$ of the nilpotent linear transformation with respect to the constructed orthonormal basis."
Lectures by Dr. Gautam Bharali on “Complex Analytic Functions from the Viewpoint of Real Analysis”

The 3-days long lecture series by Dr. Bharali started by first giving the participants a glimpse into the definition of complex analytic or holomorphic functions in terms of a power series and angle preservation. He also introduced angle preservation property of real linear transformations by expressing the transformation as a standard matrix and by providing the necessary and sufficient conditions for it to be so. This was followed by the definition of holomorphic mapping where the mapping was into $\mathbb{R}^2$ and of class $C^1$ and the derivative of the mapping acted as complex linear. This further led to the Cauchy-Riemann conditions.

The second day began with the concepts of derivatives in higher dimensions graphically and theoretically. On the third and final day, Dr. Bharali began by telling about the theorem which gives three equivalent conditions for a function to be holomorphic. Examples to determine whether a given mapping was holomorphic or not followed. The lecture was concluded with a theorem about conformal maps.

Throughout the lecture series, the concept of complex analysis was introduced in an all-together different light, especially for the undergraduate participants. This lecture series has helped the participants to look at the topic through an enriching and interesting angle.

Lectures by Prof. B. Ramakrishnan “On Decimal Expansion of Real Numbers”

He started his lecture by giving an example:

$$1.98765012... = 1 + (9/10) + (8/100) + (7/1000) + \ldots$$

with the help of which he wanted to explain that “any real number can be written to any specified base”.

He introduced the term “b-ary expansion”, which is basically the expansion of any real number with respect to any specified base $b$, a positive integer, where we fix $b \geq 2$. That is, if $c$ is any real number and $b \geq 2$ is any base then,

$$c = a.(a_1)(a_2)(a_3)\ldots \text{ where these } (a_1), (a_2), (a_3), \ldots \text{ belong to } \{1, 2, 3, \ldots, b-1\}$$

He then also stated a theorem that: "every possible b-ary expansion converges to a positive real number and conversely, every positive real number has a b-ary expansion". He then gave some of the examples based on the above theorem. He then shared another theorem which was called the "Non-Uniqueness of the b-ary Expansions of a Real Number". He also shared "when does a real number have a unique b-ary expansion?" He then gave the definition of the term: "Eventually Periodic b-ary Expansion" and the period of that expansion.

On the second day, he gave examples of "purely periodic expansions" and "eventually periodic expansions" and also shared a theorem with proof that: "a positive real number is eventually periodic if and only if that number is a rational number". He described some examples of b-ary expansions of some real numbers with respect to the base 10, with the help of which he explained the periods of those expansions. He then explained how these periods could be linked with the multiplicative group $U(n)$.

On the third day, he started with a theorem which states: "if we have an irrational number lying between 0 and 1 and $B$ is any block of $n$-digits in base $b$, then there exists an integer between 1 and $b^n(2n+1)$ such that if we multiply that irrational number by this integer then the b-ary expansion of the resulting real number will have infinitely many blocks of $B$". He then gave "Weaker Version" of the above theorem. He concluded his lecture with the proof of this theorem.
(iv) TUTORIALS:
Towards the end of each day, there was a tutorial session which was really beneficial for students as they could clear their doubts from the speakers. It was a good opportunity for the students to interact with the renowned mathematicians. In this session, the speakers gave some problems to students related to their lecture. Students devoted this time in solving these problems and they could understand the concepts better.

B : Summary of Feedbacks

“It was indeed an honour to be a part of the Workshop held at Miranda House. I am really grateful for giving us this opportunity to enhance our knowledge through the valuable talks by such eminent mathematicians. The lectures indeed helped us to look into a broader horizon of our subject. Looking forward to such workshops in future.”

Hiya Saharia
Daulat Ram College
III year

“Ma’am! I kindly want to tell you that the workshop you organised for us was really very good and knowledgeable. All the three lectures organised for the three days were very knowledgeable. The lecture on ”Inequalities in Complex Analysis” by Professor Gadadhar Misra was nice. We got to know many things like another proof for Cauchy-Schwarz inequality, new applications of inner product space and many more.

Again lecture by Dr. Gautam Bharali on ”Complex-Analytic functions from the viewpoint of Real Analysis” was also very very knowledgeable. We learnt many new things from him like what do we mean by complex and real analytic functions, what are angle-preserving maps, what are holomorphic functions(two other definitions other than what we are going to study in our next semester), when do we call functions of two or more variables differentiable (theoretically as well as graphically that I probably didn’t know before), proof of theorem on holomorphic functions and many more. The best thing that I observed and liked about his lecture was that he gave us tutorial questions to solve with the help of which our understanding of the concept became more clear and better.

At last but not the least the lecture by Professor B Ramakrishnan on ”Decimal Expansion of Real Numbers” was also very good. We got to know about the b-ary expansions of the real numbers like when does the b-ary expansion of an irrational number have repeating blocks of 00’s(or any other repeating block) and many more.”

Praveen Manju
B.Sc.(Hons.) Math
Miranda House
3rd Year

“It first of all gave us a golden opportunity to interact with such renowned and eminent mathematicians. All the three professors were outstanding and taught us the core of how should mathematics be approached and learned. I personally found the topic of lectures by Prof. Gadadhar Misra and Dr. Gautam Bharali of more interest. But the lecture by Prof. B Ramakrishnan was equally informative “.

Karnica Jain (III Year, Maths(H) )
Miranda House
“The workshop on Analysis recently held provided deep insights into topics most of which we have not studied yet. The lectures were quite informative and full of knowledge. For me, lectures of Dr. Gautam Bharali which included holomorphic functions stood out. They were comprehensive and easy to grasp owing to sir's knowledge and right choice of words. Lectures on inequalities in complex analysis and decimal expansion of real numbers were no less enriching. Personally, I did feel disconnected a few times probably because of lack of clarity of my concepts. Altogether it was an amazing experience.”

Sumbul Moin, III year

Miranda House